**Collective Excitations (spin)**

We’ll examine density waves and spin waves (these are the excitations we see in the Heisenberg exchange interaction thing in the next folder). We’ll recall that in the Nearly Free Model, the spin oscillations were governed by the charge density oscillations. But here, since the two-particle interaction is spin-dependent, we’ll see we get differentiated behavior between spin and charge oscillations.

**Spin-Spin Oscillations**

Now let’s consider the (spin) density – (spin) density correlation function. The spin (site) density operator is:



We might as well just focus on fluctuations in the z direction alone. Then we’ll have:



and so the spin-spin correlation function would be:



We’ll opt to work out the corresponding complex time GF,



where δns(z)(**R**,τ) = ns(z)(**R**,τ) – n0s(z)(**R**,τ), and n0s(z)(**R**,τ) just equals 0 in our case, at least we’ll presume. In terms of c’s and such, we have:



and H is the equilibrium H. And could also introduce the definition,



where,



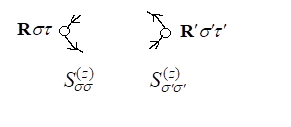
Now how do we calculate the GF? Basically I guess, we can say:



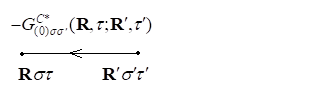
Might recall that in the nearly free file, we argued that σ had to be equal to σ´, because the potential was spin conserving. But in this case, the onsite interaction does flip spin, so we can start spin up and end up spin down. So we can’t make that same claim here.

**Real Space Rules**

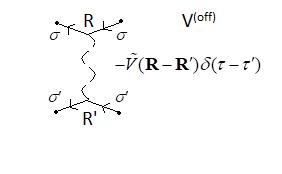
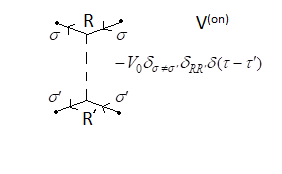
And so we might say our rules are simply, start with external points:



associating with each the diagonal spin matrix element. The rest would be just like the charge-charge rules. So connect together via the bare GF,



with powers of the interaction vertex:

And then as usual… construct all topologically distinct connected diagrams/sans vacuum bubbles. Then we have the usual stuff – each diagram gets factor of one, fermion loops get (-1). Then sum over all internal vertices/times/spins. Seems up to RPA we can treat the spin sum as reducing to factor of two for fermion loops, but with compensatory factor of (1/2) for every V(on) interaction. But don’t forget there is still an overall (-) sgn as well [see the expression above].

**Fourier Space Rules**

Re-examine what we did for the current-current correlation function, and what happened when we took the Fourier transform, and we’ll see that we have the following

expansion (remember momenta flow against arrows):

A diagram of ovals and circles

Description automatically generated with medium confidence

Construct all topologically distinct diagrams. Start with ‘dummy’ energy–momentum (k,iωn­) in the top line above the left hand vertex. Impose energy-momentum conservation at each vertex. Add (q,iν­n­) ’boost’ once cross the other vertex. Overall energy-momentum conservation will ensure that energy-momenta ‘below’ the L and R vertices will be (q,iνn­) greater than the momenta above the L and R vertices. Each vertex gets that spin matrix term above Sσσ. Each single unbroken line labeled k,iωn­ represents a factor of G0C\*(k,iωn) as before. And wavy lines give us the interaction of courrse. Fermion loops get (-1). Sum over all unconstrained momenta (including the k and k´) and unconstrained frequencies (including the iωn) with the 1/N and 1/β factor as usual. Quasi-generally, or at least up to RPA, spin sum reduces to factor of two for fermion loops with caveat that V(on) reduces fermion loops by factor of two. But don’t forget the overall [-] too.

**Spin - Spin Correlation Function Self-Energy (RPA)**

Now we’d like to work out the energy and lifetime of these excitations, which requires extraction of the self-energy. We can depict it diagrammatically as follows:

A diagram of a diagram

Description automatically generated with medium confidence

ΠC\*(s)irr(q,iνn) functions as the self-energy, and comprises all diagrams that cannot be separated by cutting through a single interaction line – but we’re doing RPA approximation here obviously. The corresponding equation reads, suprisingly (where s = ½):



Let’s see how this comes about…specifically, how we get the -½ in front of V0 and how q doesn’t even show up. We’ll start by considering the zeroth order term. Noting the spin-dependent correlation function is ΠC\*irr(RPA)σσ´ = (1/2)ΠC\*irr(RPA)δσσ´ (because the bare GF0 are diagonal in spin and because the regular ΠC\*irr(RPA) includes a sum over spins which results in a factor of two, which we have to undo to get ΠC\*irr(RPA)σσ´ whose spins are not being summed over), we have:



Now consider the first order terms. These would give us:



…top guy goes to zero since ΣSσσ is ½ - ½. Now let’s consider the second order terms. These are:



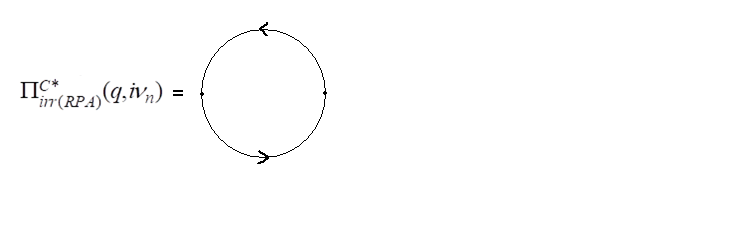
And so the pattern seems to hold. Then we have:



Analytically continuing:



And we’ll pause to note that this expression seems to follow automatically from the spin/charge separated Hamiltonian in the Interaction file. I guess that’s part of its utility. Note that poles of ΠR(s) will give dispersion spectrum, ω(k). Now let’s work out the RPA approximation to Πirr, which is all we need to evaluate the highlighted expression.



We already found this in the charge file of course. We got:



and,



Recall that in the Charge Collective Oscillations file, we evaluated this in two limits (q,ω) → 0,a nd q << ω. We get, at least for a cubic lattice dispersion relation:



So in those limits we have:



So we see that the roots of ΠR(zz)(s)RPA, in the large ω limit at least, are purely imaginary. So spin waves don’t exist as excitations of the system in this limit, at least up to RPA. This is thought to be true for relatively weak onsite repulsion V0 << band width W. But for stronger repulsion, in which case more terms would have to be included in the self-energy, it seems we have otherwise. And in any event, for strong V0 it seems spin waves must be supported by Goldstone’s theorem, because Ferromagnetism is supported, which is a broken symmetry. And recall (or foretell) by comparison that we *do* get spin-wave excitations in the Exchange model of Ferromagnetism (in next folder), and we did get them, by extension, in a previous file when we saw the Hamiltonian reduced to the Exchange model in the limit V0 >> t. So we *would* expect them here as well, if V0 is strong enough, because a strong V0 is what makes the exchange interaction important. This sort of makes sense with our results. We can only presume that our results above are for ‘relatively’ weak V0, because when it becomes super strong, the denominator of ΠR(q,0) becomes singular, and then negative. But as we’ll see in a bit, ΠR is basically the magnetic susceptibility, and it would make sense for it to switch sign like that. So basically our result is only good for V0 < 2ρF(μ). This inequality is basically the Stoner criterion for the onset of antiferromagnetic/ferromagnetic behavior anyway.